University of Misrata- Faculty of Engineering
Mechanical Engineering Department
(Heat Transfer 2)
Solution of HW2

Q1)

At 38°C $\rho = 993$ $\mu = 6.82\times10^{-4}$ $k = 0.63$ $\Pr = 4.53$

$c_p = 4180$ $\text{Re} = \dfrac{(993)(1.5)(0.0064)}{6.82\times10^{-4}} = 13{,}978$

$$h = \frac{0.63}{0.0064}(0.036)(13{,}978)^{0.8}(453)^{1/3}\left(\frac{0.0064}{0.15}\right)^{0.055} = 10{,}213\ \frac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$$

$$q = (10{,}213)\pi(0.0064)(0.15)(28) = (993)\frac{\pi}{4}(0.0064)^2(1.5)(4180)(T_e - 38)$$

$= 862.5\ \text{W}$

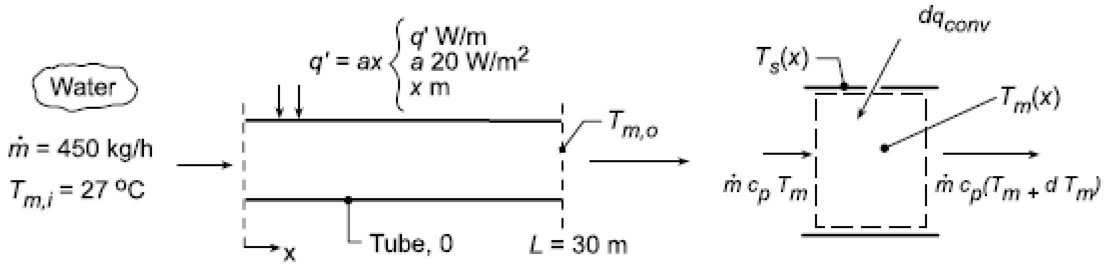
$T_e = 42.31^\circ\text{C}$

Q2)

KNOWN: Internal flow with prescribed wall heat flux as a function of distance.

FIND: (a) Beginning with a properly defined differential control volume, the temperature distribution, $T_m(x)$, (b) Outlet temperature, $T_{m,o}$, (c) Sketch $T_m(x)$, and $T_s(x)$ for fully developed *and* developing flow conditions, and (d) Value of uniform wall flux q_s'' (instead of $q_s' = ax$) providing same outlet temperature as found in part (a); sketch $T_m(x)$ and $T_s(x)$ for this heating condition.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Incompressible flow.

PROPERTIES: *Table A.6*, Water (300 K): c_p = 4.179 kJ/kg·K.

ANALYSIS: (a) Applying energy conservation to the control volume above,

$$dq_{conv} = \dot{m}c_p dT_m \tag{1}$$

where $T_m(x)$ is the mean temperature at any cross-section and $dq_{conv} = q' \cdot dx$. Hence,

$$ax = \dot{m}c_p \frac{dT_m}{dx}. \tag{2}$$

Separating and integrating with proper limits gives

$$a\int_{x=0}^{x} x\,dx = \dot{m}c_p \int_{T_{m,i}}^{T_m(x)} dT_m \qquad T_m(x) = T_{m,i} + \frac{ax^2}{2\dot{m}c_p} \tag{3,4}$$ <

(b) To find the outlet temperature, let x = L, then

$$T_m(L) = T_{m,o} = T_{m,i} + aL^2/2\dot{m}c_p. \tag{5}$$

Solving for $T_{m,o}$ and substituting numerical values, find

$$T_{m,o} = 27^\circ C + \frac{20\,W/m^2\left(30\,m^2\right)}{2\left(450\,kg/h/(3600\,s/h)\right)\times 4179\,J/kg\cdot K} = 27^\circ C + 17.2^\circ C = 44.2^\circ C.$$ <

(c) For *linear wall heating*, $q_s' = ax$, the fluid temperature distribution along the length of the tube is quadratic as prescribed by Eq. (4). From the convection rate equation,

$$q_s' = h(x)\cdot \pi D\left(T_s(x) - T_m(x)\right) \tag{6}$$

For fully developed flow conditions, h(x) = h is a constant; hence, $T_s(x) - T_m(x)$ increases linearly with x. For developing conditions, h(x) will decrease with increasing distance along the tube eventually achieving the fully developed value.

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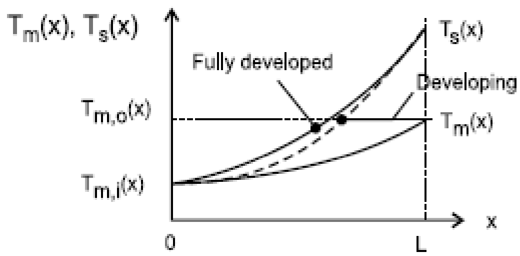


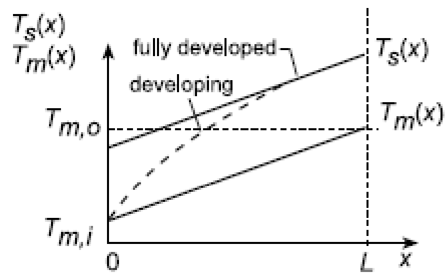
(d) For *uniform wall heat flux heating*, the overall energy balance on the tube yields

$$q = q_s''\pi DL = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right)$$

Requiring that $T_{m,o} = 44.2°C$ from part (a), find

$$q_s'' = \frac{(450/3600)\,kg/s \times 4179\ J/kg \cdot K\,(44.2 - 27)K}{\pi D \times 30\,m} = 95.3/D\ W/m^2 \qquad <$$

where D is the diameter (m) of the tube which, when specified, would permit determining the required heat flux, q_s''. For uniform heating, Section 3.3.2, we know that $T_m(x)$ will be linear with distance. $T_s(x)$ will also be linear for fully developed conditions and appear as shown below when the flow is developing.



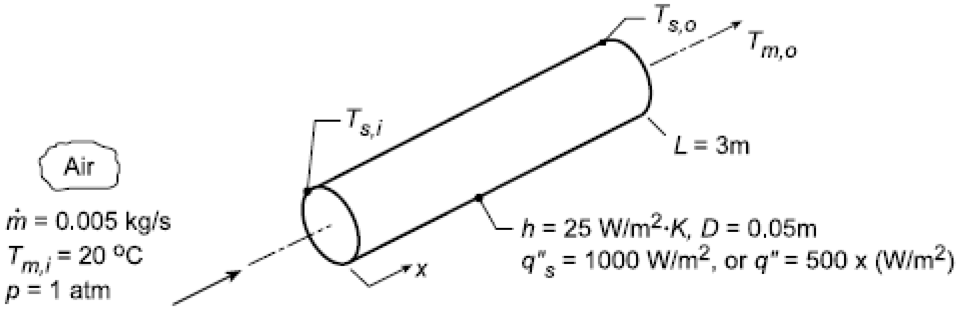
COMMENTS: (1) Note that c_p should be evaluated at $T_m = (27 + 44)°C/2 = 309$ K.

Q3)

KNOWN: Air inlet conditions and heat transfer coefficient for a circular tube of prescribed geometry. Surface heat flux.

FIND: (a) Tube heat transfer rate, q, air outlet temperature, $T_{m,o}$, and surface inlet and outlet temperatures, $T_{s,i}$ and $T_{s,o}$, for a uniform surface heat flux, q_s''. Air mean and surface temperature distributions. (b) Values of q, $T_{m,o}$, $T_{s,i}$ and $T_{s,o}$ for a linearly varying surface heat flux $q_s'' = 500x$ (m). Air mean and surface temperature distributions, (c) For each type of heating process (a & b), compute and plot the mean fluid and surface temperatures, $T_m(x)$ and $T_s(x)$, respectively, as a function of distance; What is effect of four-fold increase in convection coefficient, and (d) For each type of heating process, heat fluxes required to achieve an outlet temperature of $T_{m,o} = 125°C$; Plot temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Fully developed conditions in the tube, (2) Applicability of Eq. 8.36, (3) Heat transfer coefficient is the same for both heating conditions.

PROPERTIES: *Table A.4*, Air (for an *assumed* value of $T_{m,o} = 100°C$, $\overline{T}_m = (T_{m,i} + T_{m,o})/2 = 60°C = 333$ K): $c_p = 1.008$ kJ/kg·K.

ANALYSIS: (a) With constant heat flux, from Eq. 8.39,

$$q = q_s''(\pi DL) = 1000\,\text{W/m}^2(\pi \times 0.05\,\text{m} \times 3\,\text{m}) = 471\,\text{W}. \qquad (1)$$

From the overall energy balance, Eq. 8.37,

$$T_{m,o} = T_{m,i} + \frac{q}{\dot{m}c_p} = 20^\circ\text{C} + \frac{471\,\text{W}}{0.005\,\text{kg/s} \times 1008\,\text{J/kg}\cdot\text{K}} = 113.5^\circ\text{C} \qquad (2) <$$

From the convection rate equation, it follows that

$$T_{s,i} = T_{m,i} + \frac{q_s''}{h} = 20^\circ\text{C} + \frac{1000\,\text{W/m}^2}{25\,\text{W/m}^2\cdot\text{K}} = 60^\circ\text{C} \qquad (3) <$$

$$T_{s,o} = T_{m,o} + q_s''/h = 113.5^\circ\text{C} + 40^\circ\text{C} = 153.5^\circ\text{C} \qquad <$$

From Eq. 8.40, (dT_m/dx) is a constant, as is (dT_s/dx) for constant h from Eq. 8.31. In the more realistic case for which h decreases with x in the entry region, (dT_m/dx) is still constant but (dT_s/dx) decreases with increasing x. See the plot below.

(b) From Eq. 8.38,

$$\frac{dT_m}{dx} = \frac{500x(\pi D)}{\dot{m}c_p} = \frac{500x\,\text{W/m}^2(\pi \times 0.05\,\text{m})}{0.005\,\text{kg/s} \times 1008\,\text{J/kg}\cdot\text{K}} = 15.6x\,\text{K/m}. \qquad (4)$$

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Integrating from x = 0 to L it follows that

$$T_{m,o} = T_{m,i} + 15.6\int_0^3 x\,dx = 20^\circ C + 15.6\left.\frac{x^2}{2}\right|_0^3 = 20^\circ C + 70.2^\circ C = 90.2^\circ C. \qquad (5)$$ <

The heat rate is

$$q = \int q_s'' dA_s = 500\left(\pi \times 0.05\,m\right)\int_0^3 x\,dx = 78.5\left.\frac{x^2}{2}\right|_0^3 = 353\,W$$ <

From Eq. 8.28 it then follows that

$$T_s = T_m + q_s''/h = T_{m,i} + 15.6\frac{x^2}{2} + \frac{500}{25}x = 20^\circ C + 7.8x^2 + 20x \qquad (6)$$

Hence, at the inlet (x = 0) and outlet (x = L),

$$T_{s,i} = T_{m,i} = 20^\circ C \qquad \text{and} \qquad T_{s,o} = 150.2^\circ C$$ <

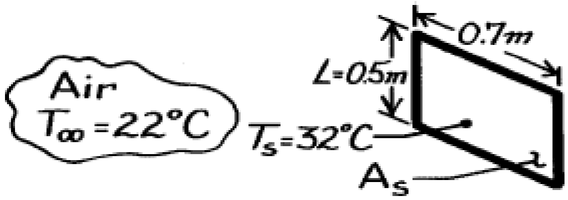
Note that (dT_s/dx) and (dT_m/dx) both increase linearly with x, but $(dT_s/dx) > (dT_m/dx)$.

Q4)

KNOWN: Oven door with average surface temperature of 32°C in a room with ambient air at 22°C.

FIND: Heat loss to the room. Also, find effect on heat loss if emissivity of door is unity and the surroundings are at 22°C.

SCHEMATIC:



ASSUMPTIONS: (1) Ambient air is quiescent, (2) Surface radiation effects are negligible.

PROPERTIES: *Table A-4*, Air (T_f = 300K, 1 atm): $\nu = 15.89 \times 10^{-6}\ m^2/s$, $k = 0.0263$ W/m·K, $\alpha = 22.5 \times 10^{-6}\ m^2/s$, Pr = 0.707, $\beta = 1/T_f = 3.33 \times 10^{-3}\ K^{-1}$.

ANALYSIS: The heat rate from the oven door surface by convection to the ambient air is

$$q = \bar{h} A_s (T_s - T_\infty) \qquad (1)$$

where $\bar{h}$ can be estimated from the free-convection correlation for a vertical plate, Eq. 9.26,

$$\overline{Nu}_L = \frac{\bar{h} L}{k} = \left\{ 0.825 + \frac{0.387 Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}} \right\}^2 . \qquad (2)$$

The Rayleigh number, Eq. 9.25, is

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha} = \frac{9.8\,m/s^2\,(1/300K)(32-22)K \times 0.5^3\,m^3}{15.89\times10^{-6}m^2/s \times 22.5\times10^{-6}m^2/s} = 1.142\times10^8.$$

Substituting numerical values into Eq. (2), find

$$\overline{Nu}_L = \left\{ 0.825 + \frac{0.387\left(1.142\times10^8\right)^{1/6}}{\left[1 + (0.492/0.707)^{9/16}\right]^{8/27}} \right\}^2 = 63.5$$

$$\bar{h}_L = \frac{k}{L}\overline{Nu}_L = \frac{0.0263\,W/m\cdot K}{0.5m} \times 63.5 = 3.34\,W/m^2\cdot K$$

The heat rate using Eq. (1) is

$$q = 3.34\,W/m^2\cdot K \times (0.5\times0.7)m^2\,(32-22)K = 11.7W.$$ <

Heat loss by radiation, assuming ε = 1, is

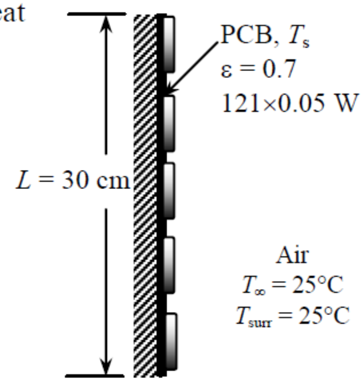
$$q_{rad} = \varepsilon A_s \sigma \left(T_s^4 - T_{sur}^4\right)$$

$$q_{rad} = 1(0.5\times0.7)m^2 \times 5.67\times10^{-8} W/m^2\cdot K^4 \left[(273+32)^4 - (273+22)^4\right] K^4 = 21.4W.$$ <

Note that heat loss by radiation is nearly double that by free convection. Using Eq. (1.9), the radiation heat transfer coefficient is $h_{rad} = 6.4\ W/m^2 K$, which is twice the coefficient for the free convection process.

Q5)

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The heat transfer from the back side of the circuit board is negligible.



Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (35+25)/2 = 30$°C are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$

$$\upsilon = 1.608\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7282$$

$$\beta = \frac{1}{T_f} = \frac{1}{(30+273)\text{K}} = 0.0033 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 35°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the height of the board, $L_c = L = 0.3$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.0033 \text{ K}^{-1})(35-25 \text{ K})(0.3 \text{ m})^3}{(1.608\times10^{-5} \text{ m}^2/\text{s})^2}(0.7282) = 2.463\times10^7$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(2.463\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7282}\right)^{9/16}\right]^{8/27}}\right\}^2 = 40.57$$

$$h = \frac{k}{L}Nu = \frac{0.02588 \text{ W/m.°C}}{0.3 \text{ m}}(40.57) = 3.50 \text{ W/m}^2\text{.°C}$$

$$A_s = (0.3 \text{ m})^2 = 0.09 \text{ m}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$

$$(121\times0.05)\text{ W} = (3.50 \text{ W/m}^2\text{.°C})(0.09 \text{ m}^2)(T_s - 25)\text{°C}$$
$$+(0.7)(0.09 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(T_s + 273 \text{ K})^4 - (25+273 \text{ K})^4]$$

Its solution is

T_s = **33.5°C**

which is sufficiently close to the assumed value in the evaluation of properties and h. Therefore, there is no need to repeat calculations by reevaluating the properties and h at the new film temperature.

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